Fall 2021 Quantum Expanders Outline

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Outline

Expander graphs are roughly families of finite graphs with small degree but high connectivity. These graphs have played a prominent role in Theoretical Computer Science. For example, one key property is that random walks across these graphs converge quickly to the stationary distribution over their vertices. This can allow efficient sampling over a finite but possibly extremely large sample space. Expanders have also seen great utility in the realms of complexity theory, such as in Dinur's proof of the celebrated PCP Theorem [5], and even classical error-correcting codes [20]. The exposition shown in [13] gives a wealth of usage examples in the context of Computer Science while [17] illustrates cases found in Pure Mathematics.

Over the past two decades, efforts to generalize these expander graphs over to the quantum realm have yielded intriguing results and applications. In particular, certain extensions known as *quantum t-tensor product expanders* have been shown to yield *approximate unitary t-designs* [12] in the sense that iterating these quantum tensor product expanders yield an efficient method of sampling from the Haar distribution over the unitary groups up to t moments. Quantum expanders find other crucial applications across quantum computation and information. Example applications include cases in physics with Hastings studying the entanglement entropy of some gapped one-dimensional systems using quantum expanders [10], quantum error-correcting codes [4, 16], and random quantum channels [1, 6].

In this reading course, we plan to survey and understand the current literature behind classical and quantum expanders. Likely candidates for study include current survey and intergal publications detailing definitions and some explicit constructions behind classical expanders [13, 18, 15], particularly ones constructed from Cayley graphs of certain finite groups [14, 17, 21], and quantum expanders with their tensor product counterparts [2, 7, 8, 9, 11, 12, 19]. Furthermore, we may study applications to unitary t-designs [3] and other areas of interest in Quantum Information Theory.

The final product will consist of an in-depth research report or set of lecture notes on Quantum Expanders. While the goal will be to advance the student's knowledge to conduct research, we realize that a full academic paper may not be feasible given the timeframe of the independent study. The report's format will resemble the format of the final report given in the previous iteration of PHY791 of Fall 2020.

Scheduled meetings will be once every week for at least one and a half hours. The day will be decided based on the instructor's schedule once the semester starts. The meetings will be Zoom or in-person. Every week homework in the form of problem sets or reading material will be assigned by the instructor. It will be the student's job to complete those tasks for discussion the following week. Grading will be based on the default three credit option. Work will be based on the student's understanding of the material and the quality of the aforementioned final product. This will be up to the discretion of the instructor.

References

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